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ABSTRACT

This study examined a selected sample of fourth-grade students who had been previously identified as good or poor problem solvers. The pupils were compared on variables considered as "reference tests" for Verbal, Induction, Numerical, Word Fluency, Memory, Spatial Visualization, and Perceptual Speed abilities. The data were compiled to indicate which intellectual abilities are most related to mathematical problem-solving performance. The results indicated that Reasoning, Verbal, and Numerical abilities are important for mathematical problem solving. Inductive Reasoning was thought to play the most significant role in discriminating between the good and poor problem solvers of the study. The entire battery of 19 tests discriminated between good and poor problem solvers with 92.2% accuracy, and some reduced models gave even higher accuracy. Several tables detailing results of the 19 tests and an extensive reference list are included in this report. (MP)

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A Comparison of the Intellectual Abilities of Good and Poor Problem Solvers: An Exploratory Study

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Abstract

Students classified as good or poor problem solvers were selected from a sample of 179 fourth-grade students. These students were then compared on variables which were "reference" tests for Verbal, Induction, Numerical, Word Fluency, Memory, Spatial Visualization or Perceptual Speed abilities.

The mean differences for the good and poor problem solvers were significant at the .01 level for Figure Matrix and Number Exclusion (Induction variables), Vocabulary and Word Group Naming (Verbal ability variables) and Mathematics Computation (Numerical ability variable). Stepwise regression procedures indicated that the entire battery of intellectual variables accounted for 42.7% of the problem solving variance. Furthermore, this battery discriminated between the two groups with 90.8% accuracy. However, reduced models, such as the one containing only Figure Matrix, Number Exclusion, and Mathematics Computation, discriminated with 92.2% accuracy.

A COMPARISON OF THE INTELLECTUAL ABILITIES OF GOOD AND POOR PROBLEM SOLVERS: AN EXPLORATORY STUDY

This study investigated intellectual variables associated with the solving of non-routine mathematical problems. Students identified as good or poor mathematical problem solvers were selected from a sample of 179 fourth-grade students. These students were then compared on a battery of 19 intellectual variables.

Immediate Background

In the November 1978 issue of the Journal for Research in Mathematics Education, the investigator presented the results of a study which investigated relationships between mathematical problem solving performance and intellectual abilities. More specifically, the investigator attempted to identify a structure of intellectual abilities related to mathematical problem solving performance.

Nineteen "reference" tests for intellectual abilities were administered to a sample of 179 fourth-grade children from Wisconsin, Illinois, and New York. All of these reference tests, with the exception of Mathematics Computation, (Romberg, 1975) had been selected from a battery of tests used by the Structure of Concept Attainment Abilities Project (CAA) (Harris & Harris, 1973).

The CAA investigation was conducted at the Wisconsin Research and Development Center for Cognitive Learning to determine a structure of concept attainment abilities. During 1970 and 1971, batteries of reference tests for cognitive abilities and tests to measure attainment or achievement of mathematics,

social studies, science, and language arts concepts were administered by the CAA staff to samples of fifth-grade males and females. Factor-analytic procedures were used to identify a basic cognitive-abilities structure and to determine relationships between concept learning in the four selected school subjects and cognitive abilities. Harris and Harris (1973) summarized the results in the following manner:

> We conclude that seven latent cognitive abilities underlie the test batteries that were studied and that these are the same for boys and girls. The seven abilities are: Verbal, Induction, Numerical, Word Fluency, Memory, Perceptual Speed, and Simple Visualization. The first six are the seven Primary Mental Abilities of the Thurstones. The seventh is similar to the Thurstone's Closure One but we prefer to call it Simple Visualization. (p. 169)

Furthermore, the CAA staff found that

1. achievement in science and social studies was related to three abilities - Verbal, Induction, and Memory.
2. achievement in language arts and mathematics was related to three abilities - Numerical, Word Fluency, and Memory.
3. two abilities - Perceptual Speed and Simple Visualization - seemed not to be related to achievement in these four subject - matter fields. (Harris & Harris, 1973, p. 195)

The results of Harris and Harris (1973) suggested the existence of a somewhat stable intellectual structure of Verbal, Numerical, Reasoning,

Spatial, Perceptual Speed and Memory factors. Since Meyer (1978) was interested in identifying a structure of intellectual abilities related to mathematical problem solving, she selected reference tests from the CAA battery that she hypothesized as being related to problem solving. Table 1, which lists the 19 reference tests administered to the sample, indicates the intellectual abilities hypothesized for the respective reference tests and gives the source of each test.

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Place Table 1 about here
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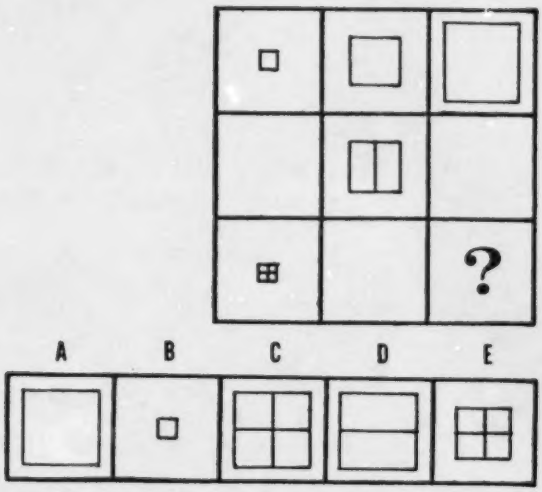
Instruments

A short description of each of the 19 reference tests was given in the Journal for Research in Mathematics Education (Nov. 1978); therefore, only the reference tests which played significant roles in this present study are discussed again here. Samples of superitems from the Romberg-Wearne Mathematical Problem Solving Test, (1975) the problem solving criterion test for this study, are also presented.

Figure Matrix. In this test the subject is to infer two spatial relations (across and down), combine them, and select from five choices the figure that belongs in the cell with the question mark.

Example:



Mathematics Computation. This test consists of finding the answers to problems involving: addition, subtraction, place value, ordering, finding the missing number, and representing parts of a whole.

Number Exclusion. In this test the subject is to infer a class from three of four given exemplars, and to indicate the one exemplar that is incorrect for that class.

Example: A. 5 B. 75 C. 750 D. 885

Vocabulary. In each item of this test the subject is to select from four exemplars a synonym for the underlined word in a phrase.

Word Group Naming. In each item of this test four exemplars of a class are given. The subject must supply a name for the class.

Example: tepee nest

beehive igloo are all _____

Romberg-Wearne Mathematical Problem Solving Test. The Romberg-Wearne test differs from other problem solving tests in that it was designed to yield three scores: a comprehension score, an application score, and a problem solving score. To accomplish this, the test was composed of groups of items called superitems. Each of these 19 superitems contained a comprehension question, an application question, and a problem solving question. Two examples of superitems are given to illustrate the nature of the comprehension, application, and problem solving parts.

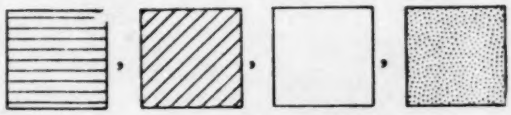
Example 1:

Jack and Bob went fishing. They both caught two kinds of fish, trout and pike. Bob caught 10 trout. Together they caught 18 trout and 10 pike. Jack caught 15 fish.	(Item Stem)
How many trout did Bob catch?	(Comprehension Question)
How many fish did they catch altogether?	(Application Question)
How many pike did Bob catch?	(Problem Solving Question)

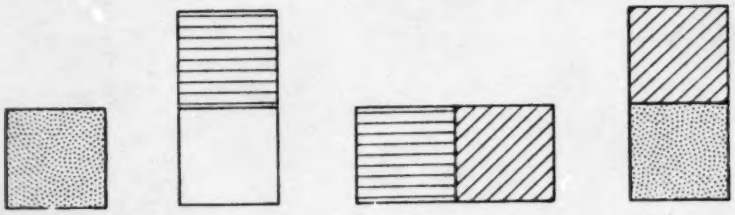
The comprehension question of example 1 ascertained whether a child understood the information given explicitly in the item stem. The application question can be answered by the direct application of the data given in the

item stem, that is, merely by adding 18 and 10. In general, each application question of the superitems assessed a child's master of a prerequisite concept or skill of the problem-solving question; the application question was a fairly straightforward application fo some rule or concept to a situation. Each problem-solving question posed a question whose solution was not immediately available, that is, a situation that did not lend itself to an immediate application of a rule or algorithm (Romberg & Wearne, 1975). The problem solving questions were not the typical textbook-type, problem situations for fourth-grade children.

Example 2:

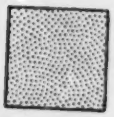
Each of the four blocks,  , covers a number of chips.

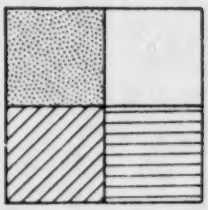
The number below these pictures tell how many chips are covered by the block combinations. (Item Stem)



2 8 10 9

7

How many chips are covered by  ? (Comprehension Question)

How many chips are covered by  ? (Application Question)

How many chips are covered by ? (Problem Solving Question)

Means, Standard Deviations, and Reliabilities of Instruments

The GITAP Program (Baker, 1969) was used to obtain means, standard deviations, and Hoyt analysis of variance reliability estimates for each of the 19 reference tests for intellectual abilities and three parts of the Romberg-Wearne Mathematical Problem Solving Test. These statistics are presented in Table 2.

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Place Table 2 about here
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Ten of the estimates were greater than or equal to .80; two estimates were greater than .90. The reliability of the application part of the problem solving test was .69; however, the reliabilities of the comprehension and problem solving parts were only .48 and .59 respectively.

Related Literature

The continued challenge of mathematical problem solving for both pupils and teachers has led to numerous research studies. Children who are good problem solvers are being asked to solve problems out loud in order that their successful processes may be analyzed. Investigators are studying formal analytic approaches to the teaching of problem solving. Characteristics and abilities of good problem solvers are being studied.

Some inroads are being made in the area of intellectual abilities and mathematical problem solving. For example, literature yields considerable evidence that intelligence is significantly related to mathematical problem solving (Suydam & Weaver, 1977; Dodson , 1972). In her study, An Investigation of Selected Mental, Mathematical, Reading, and Personality Assessments

as Predictors of High Achievers in 6th Grade Mathematical Problem Solving, Talton (1973) suggested that total intelligence is the greatest individual contributor to high achievement in verbal problem solving. She also suggested that Verbal ability is useful for differentiating high and low achievers.

Several investigators have found that computation and/or reading are related to mathematical problem solving (Engelhart, 1931; Martin, 1963; Balow, 1964; Thompson, 1967). However, Suydam and Weaver (1977) commented about the relationships of computation, reading, and mathematical problem solving:

> Computational difficulties appear to be a major deterrent to finding correct answers when solving problems, with reading a secondary cause of difficulty.
>
> Higher levels of problem-solving ability are often associated with higher levels of computational and reading ability, but much of this apparent relationship may be the result of the correlation of these abilities with IQ. [pp. 41-42]

Some investigators have found a positive relationship between general reasoning ability and mathematical problem solving. Alexander (1960) reported that general reasoning ability was significantly related to problem solving. Dodson (1972) concluded from his analyses that all of the variables which measured reasoning ability, both verbal and non-verbal, were strongly related to problem solving ability with the two mathematically oriented reasoning variables being the strongest discriminators among ability groups. Werdelin (1966) commented:

> Problem solving in mathematics depends primarily on the general reasoning factor R , according to the results of this study. Only to a somewhat smaller extent are factors like the deductive reasoning factor D and the numerical factor N of importance. This is a result which is closely related to the very nature of mathematics problem solving. A problem is a task which involves several elements which shall be combined in the solution. The elements may be taken from various fields, such as the verbal one, the numerical one, the visual-perceptual one, etc. Therefore it is to be expected that these problems are loaded on the R factor as it is interpreted in the above. [p. 13]

Although definitions of intelligence and general reasoning depend upon the tests used to measure these abilities, past literature suggested significant relationships between intelligence and/or general reasoning and mathematical problem solving performance.

One current trend in mathematics education is to distinguish between textbook type problems and process or non-routine type problems. The textbook type problem is generally a verbal description of a situation to which some specific basic operation applies, whereas, the process or non-routine type problem is a situation which is usually more challenging. The non-routine problem cannot be solved by simple recall or by the standard use of a computational algorithm. Some time needs to be spent in searching for a solution.

The majority of past investigators of mathematical problem solving used scores from standardized problem solving test such as the Iowa Tests of Basic Skills (1971), the Stanford Achievement Test (1965) or the

Metropolitan Achievement Test, (1970) as criterion measures. A large number of the problems contained in these tests are textbook type problems. These so called problem solving situations are frequently merely computational exercises or applications of some learned concepts for many students. Obvsiously, results such as the following of Chase (1960) were found:

> From the results of the method of analysis used in this study, it appears that, in the population of which the present subjects are a sample, in predicting a student's ability to solve verbal problems in arithmetic, as sources of basic data one may turn with considerable confidence to that pupil's ability in the mechanics of computation, his comprehension of the principles which underly the number system, and the extent to which he notices specific items of information when he reads. (p. 10)

Students should be able to solve textbook type problems, nonetheless, they also need experiences in situations for which no routine process has been learned previously. Students should experience problems which require creativity and originality. They should spend some time in reflective thinking and searching for solutions.

If teachers are expected to help their student with solving non-routine problems, they need suggestions from mathematics educators. Therefore, more research, such as that of Dodson (1972), is needed to determine those intellectual abilities which are related to solving non-routine type problems successfully.

This Exploratory Study

The mean scores for Comprehension, Application and Problem Solving of the Romberg-Wearne test administered to Meyer's sample (1978)(see Table 2) showed that understanding the information given in the item stems and mastery of prerequisite mathematics concepts and computational skills of the problem solving questions did not guarantee success with the problem solving questions. A closer inspection of the frequency scores of the problem solving test indicated that 132 subjects appeared to comprehend the data given in the Example 1 (p. 5), 105 subjects found that 28 fish were caught altogether, but only 37 subjects found that Bob caught 3 pike. For Example 2 (p. 6), 162 subjects answered the comprehension question correctly, 62 answered the application question, but only six subjects were able to find the answer to the problem solving question.

The scores for the three parts of the problem solving test and related literature on mathematical problem solving suggested the following questions:

1. If students understand the item stem of a problem, have mastered the necessary mathematics concepts and computational skills, why do they have difficulty solving the problem?
2. Do the reference tests for intellectual abilities administered to the sample provide any answers to the first question? Could these reference tests be used to discriminate between those students who performed well on all three parts of the Romberg-Wearne test and those who performed well on the comprehension and application parts but poorly on the problem solving questions?

Subjects

Since the means of Comprehension (13.52) and Application (9.72) were somewhat high and the standard deviations for these variables were small for the sample, the investigator decided to consider students whose scores were above the respective means as high achievers of these two parts. Subjects who had seven or more problems correct for the Problem Solving questions were classified as good problem solvers. Those who scored below seven problems correct were classified as poor problem solvers. This criterion for the Problem Solving questions was determined on the basis of the shape of the frequency curve, an inspection of the problem solving questions, and the low mean for Problem Solving (3.47). Therefore, the following two groups were investigated in these exploratory analyses:

1. High-High-High Group (HHH)

   Those students whose scores were above the means for the comprehension and application scores and who had seven or more correct for the problem solving questions were classified into this group. Sixteen subjects satisfied this criterion.

2. High-High-Low Group (HHL)

   Those subjects whose scores were above the means for the comprehension and application scores and who had fewer than seven correct for the problem solving questions were classified into this group. Sixty subjects satisfied this criterion.

Analyses

The hypothesis that $\mu_{HHH} = \mu_{HHL}$ was tested for each of the 19 reference tests. The means, standard deviations, and F - ratios, are given in Table 3. Significant F - ratios were found for Figure Matrix, Mathematics Computation, Number Exclusion, Vocabulary, and Word Group Naming at the .01 level. The mean differences for Number Series, Omelet, Picture Group Name Selection, Remote Completion, Seeing Trends, and Spatial Relations were significant at the .05 level.

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Place table 3 about here

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Since significant F-ratios were found, the investigator was interested in further characterizing differences between the HHH and HHL groups. For this purpose, a stepwise regression procedure (Health Sciences Computing Facility, 1969) in which the Problem Solving Score was the dependent variable and the 19 reference tests for intellectual abilities were independent variables, was used.

Because Figure Matrix had the largest F-ratio (Table 3), it was entered first into the stepwise regression procedure. After Figure Matrix, Mathematics Computation accounted for the largest amount of variance in the problem solving scores, consequently, it was selected second by the procedure and was entered with Figure Matrix. Number Exclusion was the third variable entered. The regression procedure continued until 18 of the intellectual variables had been entered. The contribution of Perceptual Speed was too small for computation, therefore, the program terminated after 18 variables.

These 18 variables accounted for 42.7% of the total problem solving variance. Table 4 presents the sequence of entrances of the variables, multiple

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Place Table 4 about here

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R's, the increase in variance accounted for by each variables entered, and F-values to enter. Only Figure Matrix, Mathematics Computation, and Number Exclusion had significant F-values to enter at the .05 level.

Because stepwise regression procedures depend highly upon the variables entered first and since the mean differences for Word Group Naming, Mathematics Computation, Number Exclusion, and Vocabulary were also significant at the .01 level for the HHH and HHL groups (see Table 3), each of these variables were entered as the first variable in the stepwise procedures.

When Word Group Naming was entered first it contributed 15.5% to the variance in the problem solving scores. After Word Group Naming as the first entrant, other significant contributions ($p < .05$) were made by Number Exclusion (8.7%), Mathematics Computation (4.8%), and Figure Matrix (4.5%).

When Mathematics Computation was entered first, its contribution to the total variance in the problem solving scores was 12.2%. After Mathematics Computation as the first entrant, other signficiant contributions ($p < .05$) were made by Figure Matrix (12.8%) and Number Exclusion (6.2%).

Vocabulary as the first entrant contributed 12.3% to the problem solving variance. After Vocabulary as the first entrant, significant contributions ($p < .05$) were made by Figure Matrix (10.2%), Number Exclusion (7.2%) and Mathematics Computation (4.5%).

When Number Exclusion was the first entrant, it contributed 10.4% to the variance of problem solving. After Number Exclusion, Word Group Naming (13.8%), Mathematics Computation (4.8%), and Figure Matrix (4.5%) also contributed significantly ($p < .05$).

Next, in order to determine the contributions of each of the seven categories of intellectual abilities (see Table 1) to the problem solving performances, the respective tests for a specific ability were entered first into the stepwise procedure. For example, to measure the contribution of induction to problem solving, Figure Matrix with the largest F-ratio (table 3) was entered first. Number Exclusion, with the largest F-ratio of the remaining induction reference tests, was entered next. Number Classification followed and finally Letter Classification was entered. After all variables for induction were entered, the next variable, which accounted for the largest amount of the remaining problem solving variance, was entered. The regression procedure continued until all 19 variables had been entered.

This procedure was followed for each respective ability. The percent of variance of the problem solving score attributed to each ability and significant contributions of other variables, after the forced entrances of reference tests for a respective ability, are given in Table 5.

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Place Table 5 about here

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Discussion

The results suggest that Figure Matrix and Number Exclusion (reference tests for Induction), and Mathematics Computation (a reference test for Num erical ability), played important problem solving roles for the sample.

In almost all stepwise regression procedures, these three variables contributed significantly to the total variance in problem solving. Only when the reference tests for Spatial Visualization or Perceptual Speed were the first entrants (Table 1) did Figure Matrix not contribute significantly. Mathematics Computation did not contribute significantly when the first entrants were the reference tests for Word Fluency.

Vocabulary and Word Group Naming, reference tests for Verbal ability, also appear to be related to successful problem solving performance. As a first entrant in the stepwise procedures, Word Group Naming accounted for 15.5% of the problem solving variance and Vocabulary, as a first entrant in the procedure, accounted for 12.3% of the variance. The significant contributions of Word Group Naming and Vocabulary are suggested also in Table 5 where Verbal ability accounts for 18.4% of the variance.

Limitations

Generalizability of the results is restricted by the unequal sample sizes, the difficulty of the problem solving questions, and the high correlations between some of the intellectual variables. To explore the limitation of unequal sample sizes, coefficients of each of the stepwise regression models were used to classify subjects into either the HHH or HHL group. The following scheme appeared to be the most efficient:

If the computed $Y \geq .4$, place subject into the HHH group.

If the computed $Y < .4$, place subject inot the HHL group.

With this classification scheme, 90.8% of the subjects were placed correctly when the entire battery of intellectual variables was used. However, for the reduced model, using only Figure Matrix, Mathematics

Computation, and Number Exclusion from Table 4, the classification scheme was 92.2% effective.

Ideally, the investigator should have used another sample of fourth-grade children to evaluate the classification scheme. Nevertheless, the classification results suggested that with unequal samples, a scheme can be found which discriminates between high and low achievers. Of course, identifying the upper one-third as good problem solvers and the lower one-third as poor problem solvers, as has been done in some previous studies, would have eliminated the unequal sample limitations. However, the investigator contends that receiving a score in the upper one-third of the class does not necessarily mean that a subject is a good problem solver.

Table 6, the correlation table used in the regression analyses, indicates

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Place Table 6 about here

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many significant intercorrelations. Consequently, when a variables such as Figure Matrix entered the regression procedure, a variable such as Word Group Naming which correlated highly with it, accounted for little additional variance. Further research is needed to determine whether the significant intercorrelations are phenomena of this particular sample.

Conclusions

Like many past investigations, this exploratory study suggested that Reasoning, Verbal, and Numerical abilities are important for mathematical problem solving. Inductive Reasoning, measured by Figure Matrix and Number Exclusion and Numerical ability, determined primarily by Mathematics Computation, discriminated between the good and poor problem solvers of this investigation most effectively.

The discriminating effect of Mathematics Computation was somewhat surprising because the investigator had attempted to control for the computational skills found in the problem solving questions by requiring that subjects have scores above the mean on Part II of the Romberg-Wearne test. The following examples suggested, however, that Mathematics Computation may have required more than computational skills of fourth-grade children.

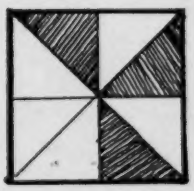
Example 1

Find the missing number

$4(\ \ ) - 6 = 22$

Example 2

What fraction is shown?



Examples 1 and 2 suggested that some items of Mathematics Computation may have been problem solving situations for some children. In fact, some items may have required reasoning ability and not computational skills.

It may have been reasoning ability which helped the good problem solvers of this study perform better on the Word Group Naming and Vocabulary tests. For Word Group Naming, students had to infer how four exemplars were related and then produce a class name. The vocabulary of fourth-graders is limited, therefore, finding a synonym for an underlined word in a phrase may have required some reasoning.

Consequently, the investigator concludes that it was reasoning ability, with inductive reasoning playing the most significant role, that discriminated between the good and poor problem solvers of this study. Moreover, she

hypothesizes that reasoning ability is significantly important for solving non-routine mathematical problems. Of course, problem solvers must have the pre-requisite skills and tools necessary for solving problems. They must be able to read problems. They must perform the required computations or be able to use a calculator for the computations. Drawing diagrams, constructing tables, organizing data, and writing equations are necessary pre-requisites. Nonetheless, all tools and experiences are of little consequence if students cannot reason from the diagrams, equations, and tables to solutions of problem solving situations.

Ordinarily, replication of an exploratory study is recommended. This is not the situation here. This investigation and past research have certainly demonstrated that reasoning ability plays a significant role in mathematical problem solving. Therefore, the investigator suggests that mathematics educators develop and experiment with instructional problem solving units which provide students with reasoning experiences.

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TABLE 1

Intellectual Abilities Hypothesized for the Population Sample, the Respective Reference Tests and Their Sources

Intellectual Abilities	Reference Tests	Sources
Verbal	Pict. Group Name Sel	Constructed by CAA[a] Staff
	Word Group Naming	Constructed by CAA Staff
	Remote Class. Comp.	Adapted from Waddle Test by CAA Staff
	Vocabulary	Iowa Tests of Basic Skills (1964)
Induction	Letter Classification	Constructed by CAA Staff
	Number Classification	Constructed by CAA Staff
	Figure Matrix	Sheridan Psychological Services (1969)
	Number Exclusion	Constructed by CAA Staff
Numerical	Mathematics Computation	Constructed by Romberg (1975)
	Number Series	Constructed by CAA Staff
	Seeing Trends	Constructed by CAA Staff
Word Fluency	Omelet	Constructed by CAA Staff
	Spelling	Iowa Tests of Basic Skills (1964)
Memory	Rem. Classes: Members	Constructed by CAA Staff
	Pict. Class Memory	Constructed by CAA Staff
Perceptual Speed	Identical Pictures	ETS Kit of Reference Tests (1962)
	Perceptual Speed	PMA 4-6 Test Battery (SRA) (1962)
Simple Visualization	Gestalt Completion	Constructed by CAA Staff
	Spatial Relations	PMA 4-6 Test Battery (SRA) (1962)

[a] CAA refers to A Structure of Concept Attainment Abilities Project (Harris & Hirris, 1973).

TABLE 2

Means, Standard Deviations and Reliability Estimates for Test Scores[a]

Test		Items	Mean	Standard Deviation	Hoyt Reliability
1	Fig Mat	20	8.92	3.91	.74
2	Gestalt	20	12.23	3.66	.75
3	Ident P	48	26.61	9.19	.95
4	Let Cla	20	13.78	3.37	.72
5	Math Comp	54	40.51	8.12	.89
6	Numb Cl	30	24.07	5.90	.91
7	Numb Ex	20	13.88	4.00	.81
8	Numb Se	20	12.84	4.15	.81
9	Omelet	20	10.32	5.01	.88
10	Per Spd	40	27.44	6.58	.89
11	Pict C M	20	15.45	3.20	.78
12	Pict G N S	20	12.08	3.07	.63
13	Rem Cl M	20	13.85	3.43	.71
14	Remote	25	12.75	4.08	.76
15	See Tnd	20	11.85	3.79	.73
16	Spat Rl	25	15.94	4.18	.76
17	Spellg	38	14.11	6.95	.87
18	Vocab	38	24.40	7.28	.89
19	Word G N	20	12.18	4.27	.80
20	Comprehension	19	13.52	2.41	.48
21	Applications	19	9.72	3.33	.69
22	Prob. Solving	19	3.47	2.40	.59

N = 179

TABLE 3

Means, Standard Deviations, and F-ratios for the 19 Reference Tests

	Tests	Group HHH[a] Number of Items	Mean	Standard Deviation	Group HHL[b] Mean	Standard Deviation	F-Ratio
1	Fig Mat	20	13.50	2.50	10.02	3.49	13.99**
2	Gestalt	20	12.19	3.90	12.42	3.79	.05
3	Ident P	48	28.62	7.92	27.47	9.68	.19
4	Let Cla	20	16.00	2.28	14.97	2.80	1.85
5	Math Comp	54	48.56	4.86	44.18	4.84	10.32**
6	Numb Cl	30	27.56	2.34	25.17	4.69	3.89
7	Numb Ex	20	17.25	2.84	14.82	2.98	8.59**
8	Numb Se	20	16.56	3.29	14.32	3.27	5.96*
9	Omelet	20	14.62	3.78	12.05	4.04	5.26*
10	Per Spd	40	30.75	4.88	28.37	6.66	1.79
11	Pict Cm	20	17.37	2.00	15.85	2.99	3.71
12	Pict GNS	20	14.44	1.67	13.15	2.34	4.26*
13	Rem Cl M	20	16.50	3.12	14.78	3.26	3.56
14	Remote	25	16.44	2.78	14.15	3.54	5.72*
15	See Tnd	20	14.87	2.31	12.38	3.74	6.42*
16	Spat Rl	25	19.12	3.28	16.85	3.82	4.73*
17	Spellg	38	29.31	4.79	27.37	5.56	1.64
18	Vocab	38	31.69	4.36	26.70	5.77	10.33**
19	Word GN	20	16.94	1.81	13.97	3.08	13.59**

[a] $N = 16$

[b] $N = 60$

*$p < .05$ **$p < .01$

TABLE 4

Figure Matrix - First Entrant
in Stepwise Regression Procedures

Step Number	Variable Entered	Multiple R	Increase in Variance	F-value to enter
1	Fig Mat	.399	.159	13.99*
2	Math Comp	.501	.092	8.95*
3	Numb Ex	.560	.062	6.54*
4	See Tnd	.588	.032	3.52
5	Vocab	.606	.022	2.40
6	Spellg	.615	.011	1.18
7	Numb Cl	.624	.011	1.26
8	Let Cla	.630	.008	.83
9	Gestalt	.634	.006	.69
10	Omelet	.643	.010	1.13
11	Spat Rl	.647	.006	.65
12	Rem Cl M	.651	.004	.44
13	Ident P	.652	.001	.13
14	Pict CM	.653	.002	.16
15	Word GN	.653	.001	.10
16	Pict GNS	.654	.000	.02
17	Numb Ser	.654	.000	.01
18	Remote	.653	.000	.02

*$p < .05$

TABLE 5

Contributions to Problem Solving Variance

First Entrants-References tests for:	Percent of Problem Solving Variance Attributed to Respective Ability	Significant Contributions ($p < .05$) of Remaining Variables	
Induction	24.5	Math Computation	7.6%
		Seeing Trends	4.0%
Numerical	18.6	Figure Matrix	11.2%
		Number Exclusion	4.8%
Verbal	18.4	Number Exclusion	8.7%
		Figure Matrix	4.1%
		Math Computation	3.8%
Spatial Visualization	7.0	Word Group Naming	11.5%
		Number Exclusion	7.8%
		Math Computation	5.1%
Word Fluency	6.7	Figure Matrix	14.9%
		Seeing Trends	6.1%
		Vocabulary	4.3%
		Number Exclusion	4.1%
Memory	6.4	Seeing Trends	11.0%
		Figure Matrix	9.4%
		Number Exclusion	4.9%
		Math Computation	4.1%
Perceptual	2.7	Word Group Naming	14.8%
		Number Exclusion	7.5%
		Math Computation	5.2%

TABLE 6

Intercorrelations of Variables[a]

	Tests	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
1	Fig Mat																					
2	Gestalt	-03																				
3	Ident P	-03	21																			
4	Let Cla	22	06	10																		
5	Math Comp	22	18	31*	13																	
6	Numb Cl	11	-02	06	20	18																
7	Numb Ex	15	-09	06	34*	07	26															
8	Numb Ser	32*	23	11	29*	18	24	25														
9	Omelet	06	41*	11	25	45*	20	17	27													
10	Per Spd	15	09	63*	04	25	-01	19	11	04												
11	Plct C H	32*	-03	13	06	16	35*	05	16	15	23											
12	Plct C N S	48*	04	05	18	14	11	06	26	06	26	30										
13	Rem Cl H	40*	-15	09	20	14	36*	11	28*	09	17	45*	40*									
14	Remote	35*	24	-15	32*	31*	15	07	39*	53*	11	16	44*	25								
15	See Tred	01	22	22	06	27	-12	14	33*	21	19	-21	08	-06	09							
16	Spat Rel	38*	28*	16	07	21	25	04	42*	28	24	28	28	21	24	07						
17	Spellg	14	22	00	33*	31*	24	02	26	52*	-04	14	20	39*	46*	15	28*					
18	Vocab	26	17	02	38*	36*	20	03	26	38*	04	28	20	45*	46*	20	13	61*				
19	Word C N	49*	25	13	39*	33*	25	07	38*	45*	12	23	49*	34*	52*	34*	42*	44*	54*			
20	Comprehension	27	-03	09	11	21	32*	01	35*	16	08	40	24	40*	31*	14	33*	31*	32*	31*		
21	Application	36*	03	16	26	29*	38*	11	37*	27	14	27	32*	50*	28*	17	19	34*	41*	40*	42*	
22	Prob. Solv.	40*	-03	05	16	35*	22	32*	27	26	15	22	23	21	27	28*	25	14	35*	39*	31*	46*

[a] Decimal points have been omitted.

* $r_{.01}(76) > .27$